INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

MATHEMATICAL METHODS
STANDARD LEVEL
PAPER 2

Tuesday 8 May 2001 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all five questions from Section A and one question from Section B.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures, as appropriate.

- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-9750G*, Sharp EL-9400, Texas Instruments TI-85.

You are advised to start each new question on a new page. A correct answer with **no** *indication of the method used will usually receive* **no** *marks. You are therefore advised to show your working. In particular, where graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.*

SECTION A

Answer all **five** *questions from this section.*

1. *[Maximum mark: 10]*

Michele invested 1500 francs at an annual rate of interest of 5.25 percent, compounded annually.

(a) Find the value of Michele's investment after 3 years. Give your answer to the nearest franc. *[3 marks]*

(b) How many complete years will it take for Michele's initial investment to double in value? *[3 marks]*

(c) What should the interest rate be if Michele's initial investment were to double in value in 10 years? *[4 marks]*

2. *[Maximum mark: 14]*

The table below represents the weights, W, in grams, of 80 packets of roasted peanuts.

Weight (W)	$80 < W \le 85$	$85 < W \le 90$	$90 < W \le 95$	$95 < W \le 100$	$100 < W \le 105$	$105 < W \le 110$	$110 < W \le 115$
Number of packets	5	10	15	26	13	7	4

(a) Use the midpoint of each interval to find an estimate for the standard deviation of the weights. *[3 marks]*

(b) Copy and complete the following cumulative frequency table for the above data.

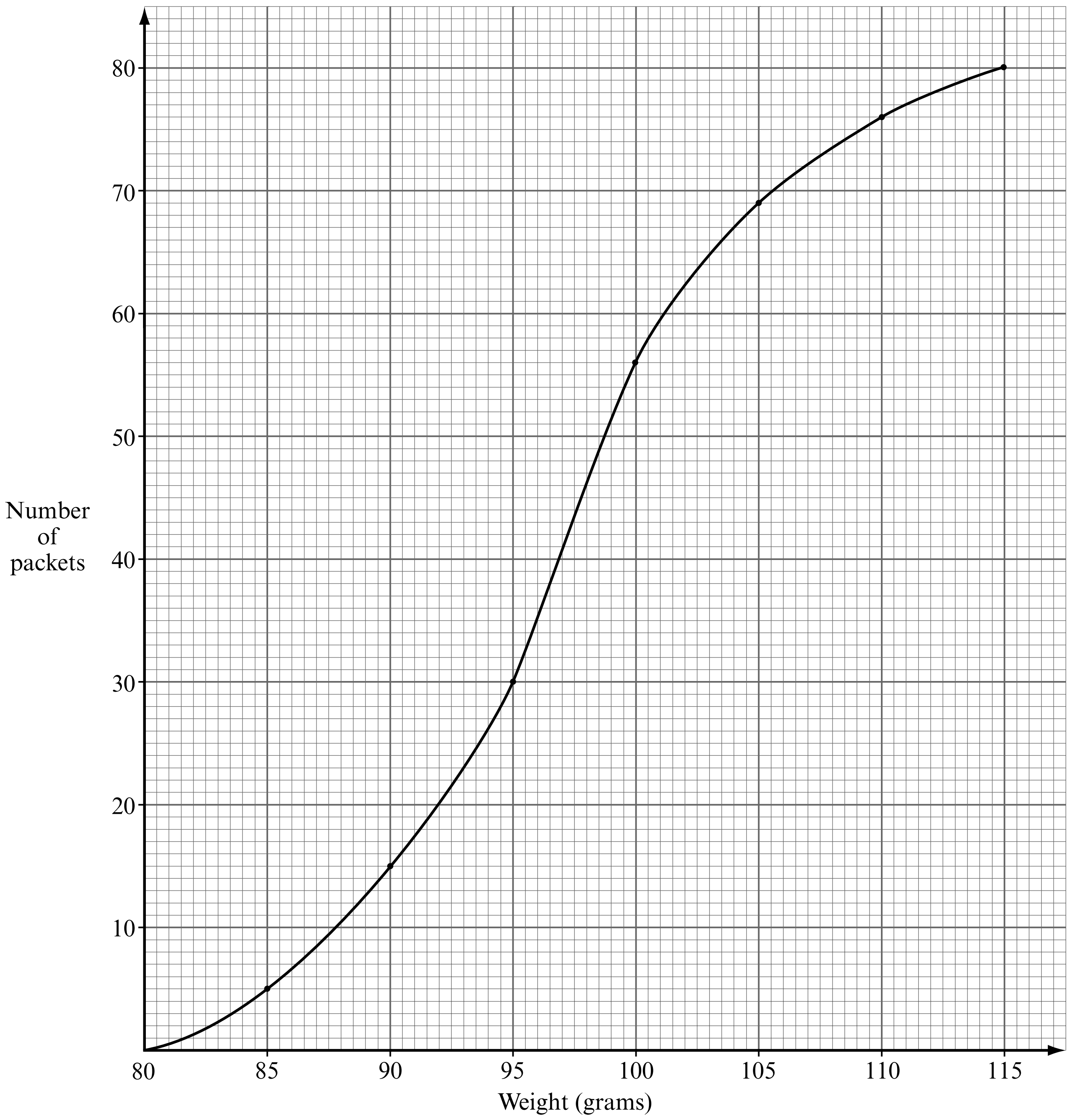
Weight (W)	$W \le 85$	$W \le 90$	$W \le 95$	$W \le 100$	$W \le 105$	$W \le 110$	$W \le 115$
Number of packets	5	15					80

[1 mark]

(This question continues on the following page)

(Question 2 continued)

(c) A cumulative frequency graph of the distribution is shown below, with a scale 2 cm for 10 packets on the vertical axis and 2 cm for 5 grams on the horizontal axis.



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(Question 2 continued)

Use the graph to estimate

(i) the median;

(ii) the upper quartile (that is, the third quartile).

Give your answers to the nearest gram. *[4 marks]*

(d) Let $W_1, W_2, \ldots, W_{80}$ be the individual weights of the packets, and let $\overline{W}$ be their mean. What is the value of the sum

$$(W_1 - \overline{W}) + (W_2 - \overline{W}) + (W_3 - \overline{W}) + \ldots + (W_{79} - \overline{W}) + (W_{80} - \overline{W})?$$ *[2 marks]*

(e) One of the 80 packets is selected at random. Given that its weight satisfies $85 < W \leq 110$, find the probability that its weight is greater than 100 grams. *[4 marks]*

3. *[Maximum mark: 14]*

In this question, a unit vector represents a displacement of 1 metre.

A miniature car moves in a straight line, starting at the point (2 , 0) . After t seconds, its position, (x , y) , is given by the vector equation

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0.7 \\ 1 \end{pmatrix}$$

(a) How far from the point (0 , 0) is the car after 2 seconds? *[2 marks]*

(b) Find the speed of the car. *[2 marks]*

(c) Obtain the equation of the car's path in the form $ax + by = c$. *[2 marks]*

Another miniature vehicle, a motorcycle, starts at the point (0 , 2) , and travels in a straight line with constant speed. The equation of its path is

$$y = 0.6x + 2 , \qquad x \geq 0 .$$

Eventually, the two miniature vehicles collide.

(d) Find the coordinates of the collision point. *[3 marks]*

(e) If the motorcycle left point (0 , 2) at the same moment the car left point (2 , 0) , find the speed of the motorcycle. *[5 marks]*

4. *[Maximum mark: 16]*

Note: Radians are used throughout this question.

Let $f(x) = \sin(1 + \sin x)$.

(a) (i) Sketch the graph of $y = f(x)$, for $0 \le x \le 6$.

(ii) Write down the x-coordinates of all minimum and maximum points of f, for $0 \le x \le 6$. Give your answers correct to **four** significant figures. *[9 marks]*

(b) Let S be the region in the first quadrant completely enclosed by the graph of f and **both** coordinate axes.

(i) Shade S on your diagram.

(ii) Write down the integral which represents the area of S.

(iii) Evaluate the area of S to **four** significant figures. *[5 marks]*

(c) Give reasons why $f(x) \ge 0$ for all values of x. *[2 marks]*

5. *[Maximum mark: 16]*

In the diagram below, the points O(0 , 0) and A(8 , 6) are fixed. The angle $\mathrm{O\widehat{P}A}$ varies as the point P(x , 10) moves along the horizontal line $y = 10$.

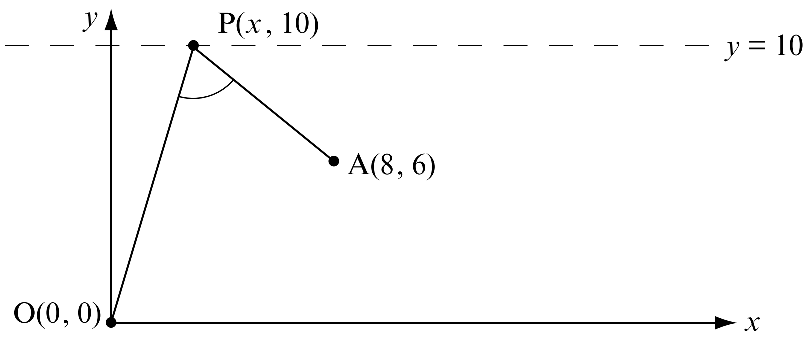


diagram not to scale

(a) (i) Show that $\mathrm{AP} = \sqrt{x^2 - 16x + 80}$.

(ii) Write down a similar expression for OP in terms of x . *[2 marks]*

(b) Hence, show that

$$\cos \mathrm{O\widehat{P}A} = \frac{x^2 - 8x + 40}{\sqrt{\{(x^2 - 16x + 80)(x^2 + 100)\}}} .$$ *[3 marks]*

(c) Find, in degrees, the angle $\mathrm{O\widehat{P}A}$ when $x = 8$. *[2 marks]*

(d) Find the positive value of x such that $\mathrm{O\widehat{P}A} = 60°$. *[4 marks]*

Let the function f be defined by

$$f(x) = \cos \mathrm{O\widehat{P}A} = \frac{x^2 - 8x + 40}{\sqrt{\{(x^2 - 16x + 80)(x^2 + 100)\}}}, \quad 0 \le x \le 15 .$$

(e) Consider the equation $f(x) = 1$.

(i) Explain, in terms of the position of the points O , A , and P , why this equation has a solution.

(ii) Find the **exact** solution to the equation. *[5 marks]*

SECTION B

Answer **one** *question from this section.*

Statistical Methods

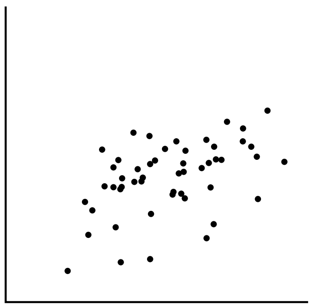
6. *[Maximum mark: 30]*

(i) The following values are the product-moment correlation coefficients for the five scatter diagrams below.

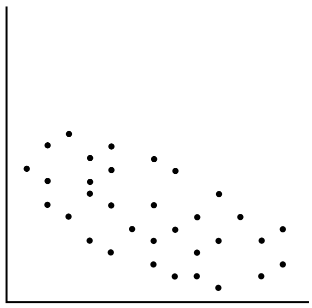
$$0.50\ ,\ -0.95\ ,\ -0.60\ ,\ 0.00\ ,\ 0.90$$

Match each scatter diagram with its corresponding correlation coefficient.

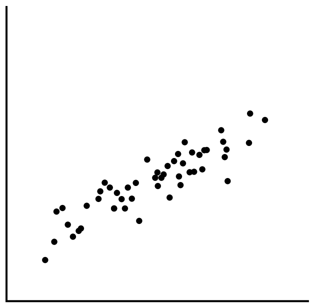
(a)



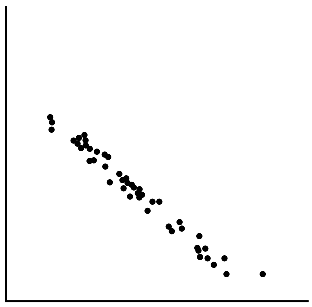
(b)



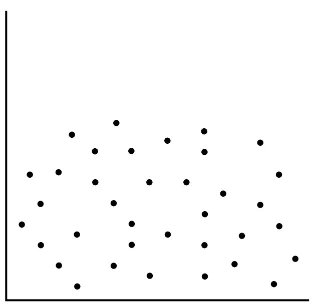
(c)



(d)



(e)



[4 marks]

(This question continues on the following page)

(Question 6 continued)

(ii) The annual advertising expenditures and sales, in dollars, for a small company are listed below.

Year	Advertising (x)	Sales (y)
1992	4000	110 000
1993	3000	65 000
1994	5000	100 000
1995	6000	135 000
1996	5000	120 000
1997	3000	90 000
1998	4000	100 000
1999	6000	140 000

(a) Find the regression line $y = a + bx$ for this data, and hence predict the annual sales that would result if 7000 dollars were spent in advertising. Give your answer to the nearest thousand dollars. *[5 marks]*

(b) If the annual advertising expenditures and annual sales figures above were converted into Japanese yen (1 dollar = 69.4017 yen) which of the following quantities would change and which would remain the same?

(i) The mean $\bar{x}$ of the advertising expenditures;

(ii) the standard deviation s_x of the advertising expenditures;

(iii) the correlation coefficient r ;

(iv) the gradient of the regression line $y = a + bx$. *[4 marks]*

(iii) Intelligence Quotient (IQ) in a certain population is normally distributed with a mean of 100 and a standard deviation of 15 .

(a) What percentage of the population has an IQ between 90 and 125? *[2 marks]*

(b) If two persons are chosen at random from the population, what is the probability that both have an IQ greater than 125? *[3 marks]*

(c) The mean IQ of a random group of 25 persons suffering from a certain brain disorder was found to be 95.2 . Is this sufficient evidence, at the 0.05 level of significance, that people suffering from the disorder have, on average, a lower IQ than the entire population? State your null hypothesis and your alternative hypothesis, and explain your reasoning. *[4 marks]*

(This question continues on the following page)

(Question 6 continued)

(iv) The manufacturer of ‘Fizz’, a popular soft drink, offered Millennium University one million dollars per year to make ‘Fizz’ the only soft drink sold on campus. A group of 140 students and teachers were consulted on whether the university should accept or reject the offer. Their responses are summarised in the following table:

	Accept the offer	Reject the offer
Students	29	59
Teachers	27	25

(a) Construct a table of expected frequencies assuming that the attitude towards the offer is independent of whether the person is a student or a teacher. Do not apply Yates’ continuity correction. *[3 marks]*

(b) Calculate χ^2 for this set of data. *[2 marks]*

(c) Based on the χ^2 test at 0.01 level of significance, which of the following conclusions may be drawn?

(i) Students are more favourable than teachers towards accepting the offer.

(ii) Students are less favourable than teachers towards accepting the offer.

(iii) Acceptance or rejection does not depend on whether the person is a student or a teacher.

Explain briefly the reasons for your answers. *[3 marks]*

Further Calculus

7. *[Maximum mark: 30]*

The function f is given by $f(x) = \dfrac{\ln 2x}{x}$, $x > 0$

(a) (i) Show that $f'(x) = \dfrac{1 - \ln 2x}{x^2}$.

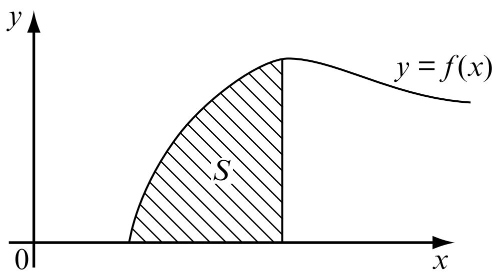
Hence

(ii) prove that the graph of f can have only one local maximum or minimum point;

(iii) find the coordinates of the maximum point on the graph of f. *[6 marks]*

(b) By showing that the second derivative $f''(x) = \dfrac{2\ln 2x - 3}{x^3}$, or otherwise, find the coordinates of the point of inflexion on the graph of f. *[6 marks]*

(c) The region S is enclosed by the graph of f, the x-axis, and the vertical line through the maximum point of f, as shown in the diagram below.



(i) Would the trapezium rule overestimate or underestimate the area of S? Justify your answer by drawing a diagram or otherwise. *[3 marks]*

(ii) Find $\int f(x)\,\mathrm{d}x$, by using the substitution $u = \ln 2x$, or otherwise. *[4 marks]*

(iii) Using $\int f(x)\,\mathrm{d}x$, find the area of S. *[4 marks]*

(This question continues on the following page)

(Question 7 continued)

(d) The Newton-Raphson method is to be used to solve the equation $f(x) = 0$.

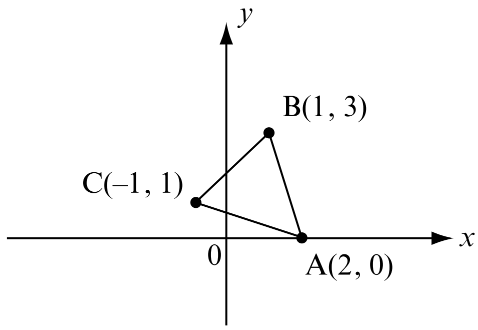
(i) Show that it is not possible to find a solution using a starting value of $x_1 = 1$. *[3 marks]*

(ii) Starting with $x_1 = 0.4$, calculate successive approximations x_2 , x_3 , . . . for the root of the equation until the absolute error is less than 0.01 . Give all answers correct to **five** decimal places. *[4 marks]*

Further Geometry

8. *[Maximum mark: 30]*

The triangle ABC has vertices A(2 , 0) , B(1 , 3) , and C(–1 , 1) .



(a) A linear transformation $\boldsymbol{T}$ maps A to A′(–12 , –6) and B to B′(–15 , –3) .

(i) Find the matrix which represents $\boldsymbol{T}$. *[4 marks]*

(ii) The triangle A′B′C′ is the image of the triangle ABC under $\boldsymbol{T}$. Find the coordinates of the point C′ . *[1 mark]*

(b) The matrix of a linear transformation $\boldsymbol{L}$ is

$$\boldsymbol{L} = \begin{pmatrix} -6 & -3 \\ -3 & 0 \end{pmatrix}$$

and the transformation $\boldsymbol{V}$ is given by

$$\boldsymbol{V} : \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \boldsymbol{L}\begin{pmatrix} x \\ y \end{pmatrix}.$$

(i) Find the point which is invariant under $\boldsymbol{V}$. *[4 marks]*

(ii) Let D be a quadrilateral whose area is a square units. What is the area of its image under $\boldsymbol{V}$? *[2 marks]*

(iii) The transformation $\boldsymbol{L}$ can be written in the form $\boldsymbol{L} = \boldsymbol{ESF}$, where $\boldsymbol{E}$ is an enlargement, $\boldsymbol{S}$ is a shear in the direction of the x-axis, and $\boldsymbol{F}$ is the reflection in the line $y = -x$. Find the matrix for each of the transformations $\boldsymbol{E}$, $\boldsymbol{S}$, and $\boldsymbol{F}$. *[4 marks]*

(This question continues on the following page)

(Question 8 continued)

(c) $\boldsymbol{W}$ is the transformation of the plane whose matrix is

$$\boldsymbol{W} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

(i) What is the image of a point (x, y) under $\boldsymbol{W}$? *[1 mark]*

(ii) Describe in words the image of the triangle ABC under $\boldsymbol{W}$. *[2 marks]*

Let $\boldsymbol{R}$ be an anticlockwise rotation of 60° about the origin, and $\boldsymbol{Q}$ a clockwise rotation of 45° about the origin.

(iii) Show that, correct to three significant figures, the matrix of the composition $\boldsymbol{QR}$ is

$$\begin{pmatrix} 0.966 & -0.259 \\ 0.259 & 0.966 \end{pmatrix}.$$ *[4 marks]*

Let $\boldsymbol{U} = \boldsymbol{WQR}$.

(iv) Give a reason why the transformation $\boldsymbol{U}$ is not an isometry. *[3 marks]*

(v) Describe, by an equation or otherwise, the set of all points (x, y) such that

$$\boldsymbol{U}\begin{pmatrix} x \\ y \end{pmatrix} = (\boldsymbol{QR})\begin{pmatrix} x \\ y \end{pmatrix}.$$ *[5 marks]*
